Exercise 15

Find an equation for the plane that

(a) is perpendicular to $\mathbf{v} = (1, 1, 1)$ and passes through $(1, 0, 0)$.

(b) is perpendicular to $\mathbf{v} = (1, 2, 3)$ and passes through $(1, 1, 1)$.

(c) is perpendicular to the line $\mathbf{l}(t) = (5, 0, 2)t + (3, -1, 1)$ and passes through $(5, -1, 0)$.

(d) is perpendicular to the line $\mathbf{l}(t) = (-1, -2, 3)t + (0, 7, 1)$ and passes through $(2, 4, -1)$.

Solution

A particular plane is specified by a normal vector $\mathbf{n}$ and a point with position vector $\mathbf{r}_0$ that lies in it. The equation for this plane comes from the fact that the dot product of $\mathbf{n}$ with any vector in the plane is zero.

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

Part (a)

The equation for the plane is

$$(1, 1, 1) \cdot (x - 1, y - 0, z - 0) = 0$$

$$1(x - 1) + 1(y - 0) + 1(z - 0) = 0$$

$$x - 1 + y + z = 0$$

$$x + y + z = 1.$$

Part (b)

The equation for the plane is

$$(1, 2, 3) \cdot (x - 1, y - 1, z - 1) = 0$$

$$1(x - 1) + 2(y - 1) + 3(z - 1) = 0$$

$$x - 1 + 2y - 2 + 3z - 3 = 0$$

$$x + 2y + 3z = 6.$$

Part (c)

The direction vector for the line is $(5, 0, 2)$, so the equation for the plane is

$$(5, 0, 2) \cdot (x - 5, y + 1, z - 0) = 0$$

$$5(x - 5) + 0(y + 1) + 2(z - 0) = 0$$

$$5x - 25 + 2z = 0$$

$$5x + 2z = 25.$$

Part (d)

The direction vector for the line is $(-1, -2, 3)$, so the equation for the plane is

$$(-1, -2, 3) \cdot (x - 2, y - 4, z + 1) = 0$$

$$-1(x - 2) - 2(y - 4) + 3(z + 1) = 0$$

$$-x + 2 - 2y + 8 + 3z + 3 = 0$$

$$-x - 2y + 3z = -13$$

$$x + 2y - 3z = 13.$$